

# Dark Sector from Interacting Canonical and Non-Canonical Scalar Fields

Rudinei C. de Souza<sup>†</sup> and Gilberto M. Kremer<sup>†§</sup>

<sup>†</sup> Departamento de Física, Universidade Federal do Paraná, Curitiba, Brazil

**Abstract.** In this work it is investigated general models with interactions between two canonical scalar fields and between one non-canonical (tachyon-type) and one canonical scalar field. The potentials and couplings to the gravity are selected through the Noether symmetry approach. These general models are employed to describe interactions between dark energy and dark matter, with the fields being constrained by the astronomical data. The cosmological solutions of some cases are compared with the observed evolution of the late Universe.

PACS numbers: 98.80.-k; 95.36.+x; 95.35.+d

§ To whom correspondence should be addressed (kremer@fisica.ufpr.br)

## 1. Introduction

It is well known that the common matter cannot explain the observed galaxy rotation curves and another type of matter is naturally necessary. Although this is an old problem [1], until the present it is not solved and the most accepted explanation is that there exists a strange matter field which interacts only gravitationally with the known matter – the so-called dark matter [2]. The recent data from the gravitational lensing effect strongly support the existence of the dark matter [3, 4].

More recently, the astronomical observations indicate that the Universe is expanding acceleratedly in the late time [5, 6]. But the standard cosmology cannot explain this observed behavior and cosmologists are looking for explanations to the current accelerated period. Until the present the most accepted idea is that there exists an exotic component with negative pressure which causes the accelerated expansion of the Universe – the so-called dark energy. It is generally described by a scalar field [7, 8] and composes the most part of the energy of the Universe in the present.

After the discovery of the accelerated expansion of the Universe, several models taking account the dark energy and dark matter – the so-called dark sector – were proposed and the most part of them consider the dark energy and dark matter as non-interacting fields. More recently, it has appeared models in the literature where it was investigated an interacting dark sector – an interesting analysis of the viability of such a interaction can be found in [9] – and the effects from a possible dark interaction upon the dynamics of galaxy clusters appear to be in agreement with the observations [10]. In the paper [11] the authors analyze the energy exchange between the dark fields. The works [12, 13, 14] propose models which use *a priori* specified scalar fields for the representation of the dark sector whereas the works [15, 16] suppose certain interactions between the dark fields and represent them by relations involving their *a priori* non-specified energy densities which are posteriorly determined. An interacting dark sector non-minimally coupled to the gravity is proposed in [17] and in the work [18] one investigates a model of dark energy interacting with neutrinos and dark matter. The growth of structures under the interaction between dark matter and dark energy was investigated in the work [19]. Also in the matter of scalar fields, the tachyon-type scalar field has received considerable attention in cosmology since it can simulate the dark energy with certain success [20, 21, 22, 23, 24, 25, 26].

In order to describe the late Universe, we consider in this work a spatially flat, homogeneous and isotropic Universe composed by an interacting dark sector and a common matter field. The dark sector will be investigated from two general models: interacting canonical scalar fields and interacting non-canonical (tachyon-type) and canonical scalar fields. The analysis starts from a general action and the potentials and couplings to the gravity are selected from the condition of existence for the Noether symmetry [26, 27, 28, 29, 30]. Each set of potentials and couplings satisfying the symmetry condition corresponds to a particular model. The field equations for some particular models resulting from the symmetry are solved and their respective

cosmological scenarios are analyzed. The cosmological solutions show that these kinds of models produce decelerated-accelerated regimes from a dynamics with energy exchange among the gravitational field and dark fields. The resulting cosmological scenarios present a good agreement with the observational data.

This paper is organized as follows: in the second section the general model of interacting canonical scalar fields is analyzed. In the subsection 2.1 the field equations are derived from a point-like Lagrangian. One selects the potentials and couplings by the Noether symmetry approach in the subsection 2.2. And in the subsection 2.3 the cosmological solutions for the most general cases are obtained. The third section treats the general model of interacting canonical and non-canonical scalar fields. In the subsection 3.1 the potentials and couplings are selected by the Noether symmetry. The field equations are derived in the section 3.2 (from a point-like Lagrangian) and the resulting equations of energy exchange are obtained. The cosmological solutions for the minimally and non-minimally coupled cases are obtained in the subsection 3.3. The conclusions about the results close the paper in the fourth section. In this work we will adopt the signature  $(+, -, -, -)$  for the metric and the natural units  $8\pi G = c = \hbar = 1$ .

## 2. Interacting canonical scalar fields

### 2.1. General action and field equations

Let us take a general action for two interacting scalar fields non-minimally coupled to the gravity of the form

$$S = \int d^4x \sqrt{-g} \left\{ [F(\phi) + G(\chi)]R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - W(\phi, \chi) \right\} + S_m, \quad (1)$$

with  $S_m = \int d^4x \sqrt{-g} \mathcal{L}_m$  being an additional action which represents a common matter field. Here  $R$  is the Ricci scalar and  $F(\phi)$ ,  $G(\chi)$  denote generic  $C^2$  functions which describe the coupling of the scalar fields to the gravitational field. Furthermore,  $V(\phi)$  is the self-interaction potential of the field  $\phi$  and  $W(\phi, \chi)$  describes the interaction between the fields  $\phi$  and  $\chi$ , including the self-interaction of the field  $\chi$ . In this action the Einstein coupling is recovered when  $F(\phi) + G(\chi) \rightarrow 1/2$ .

By varying the action (1) with respect to the metric tensor  $g_{\mu\nu}$ , we obtain the following modified Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{T_{\mu\nu}}{2(F + G)}, \quad (2)$$

where  $T_{\mu\nu} = T_{\mu\nu}^m + T_{\mu\nu}^\phi + T_{\mu\nu}^\chi$  denotes the total energy-momentum tensor related to all components of the Universe and the letters  $m$ ,  $\phi$  and  $\chi$  designate the energy-momentum tensor of the common matter and fields  $\phi$  and  $\chi$ , respectively. They are given by

$$T_{\mu\nu}^m = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}, \quad (3)$$

$$T_{\mu\nu}^\phi = \partial_\mu \phi \partial_\nu \phi - \left( \frac{1}{2} \partial_\theta \phi \partial^\theta \phi - V \right) g_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\theta \nabla^\theta) F, \quad (4)$$

$$T_{\mu\nu}^\chi = \partial_\mu \chi \partial_\nu \chi - \left( \frac{1}{2} \partial_\theta \chi \partial^\theta \chi - W \right) g_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\theta \nabla^\theta) G, \quad (5)$$

with  $\nabla_\mu$  denoting the covariant derivative.

Let us consider a flat Friedmann-Robertson-Walker (FRW) metric,  $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$  – where  $a(t)$  is the scale factor – and suppose that the scalar fields are homogeneous,  $\phi = \phi(t)$  and  $\chi = \chi(t)$ , and that the common matter is a pressureless fluid. Hence, we can write from the action (1) the point-like Lagrangian

$$\begin{aligned} \mathcal{L} = & 6a\dot{a}^2(F + G) + 6a^2\dot{a}\left(\frac{dF}{d\phi}\dot{\phi} + \frac{dG}{d\chi}\dot{\chi}\right) \\ & - a^3\left\{\frac{1}{2}\dot{\phi}^2 - V + \frac{1}{2}\dot{\chi}^2 - W\right\} + \rho_m^0, \end{aligned} \quad (6)$$

where  $\rho_m^0$  is the energy density of the common matter field at an initial instant and the point represents derivative with respect to time.

The Euler-Lagrange equations applied to the Lagrangian (6) for  $a$ ,  $\phi$  and  $\chi$  furnish

$$2\dot{H} + 3H^2 = -\frac{p}{2(F + G)}, \quad (7)$$

$$\ddot{\phi} + 3H\dot{\phi} - 6(\dot{H} + 2H^2)\frac{dF}{d\phi} + \frac{dV}{d\phi} + \frac{\partial W}{\partial \phi} = 0, \quad (8)$$

$$\ddot{\chi} + 3H\dot{\chi} - 6(\dot{H} + 2H^2)\frac{dG}{d\chi} + \frac{\partial W}{\partial \chi} = 0, \quad (9)$$

respectively. Moreover, by imposing that the energy function associated with the Lagrangian (6) vanishes, one obtains the modified Friedmann equation, i.e.,

$$E_{\mathcal{L}} = \frac{\partial \mathcal{L}}{\partial \dot{a}}\dot{a} + \frac{\partial \mathcal{L}}{\partial \dot{\phi}}\dot{\phi} + \frac{\partial \mathcal{L}}{\partial \dot{\chi}}\dot{\chi} - \mathcal{L} \equiv 0, \implies H^2 = \frac{\rho}{6(F + G)}. \quad (10)$$

In the above equations  $H = \dot{a}/a$  denotes the Hubble parameter. The set (7)-(10) are the field equations, where  $\rho = \rho_m + \rho_\phi + \rho_\chi$  and  $p = p_\phi + p_\chi$ , are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V - 6H\frac{dF}{d\phi}\dot{\phi}, \quad (11)$$

$$\rho_\chi = \frac{1}{2}\dot{\chi}^2 + W - 6H\frac{dG}{d\chi}\dot{\chi}, \quad (12)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V + 2\left(\frac{dF}{d\phi}\ddot{\phi} + 2H\frac{dF}{d\phi}\dot{\phi} + \frac{d^2F}{d\phi^2}\dot{\phi}^2\right), \quad (13)$$

$$p_\chi = \frac{1}{2}\dot{\chi}^2 - W + 2\left(\frac{dG}{d\chi}\ddot{\chi} + 2H\frac{dG}{d\chi}\dot{\chi} + \frac{d^2G}{d\chi^2}\dot{\chi}^2\right). \quad (14)$$

Now we will calculate the covariant derivative of the total energy-momentum tensor,  $\nabla_\mu T^{\mu\nu} = \nabla_\mu T_m^{\mu\nu} + \nabla_\mu T_\phi^{\mu\nu} + \nabla_\mu T_\chi^{\mu\nu}$ , in order to analyze the energy exchange among the fields. Firstly, the computation of the quantity  $\dot{\rho} + 3H(\rho + p)$  for the energy densities (11) and (12) and their respective pressures leads to

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\frac{\partial W}{\partial \phi}\dot{\phi} + \frac{(dF/d\phi)\dot{\phi}}{F + G}\rho, \quad (15)$$

$$\dot{\rho}_\chi + 3H(\rho_\chi + p_\chi) = \frac{\partial W}{\partial \phi} \dot{\phi} + \frac{(dG/d\chi)\dot{\chi}}{F+G} \rho, \quad (16)$$

where (8), (9) and (10) were used for the simplifications. The equations (15) and (16) are the same that those resulting from the covariant derivative of the energy-momentum tensors for the scalar fields  $\phi$  and  $\chi$  ( $\nabla_\mu T_\phi^{\mu\nu}$  and  $\nabla_\mu T_\chi^{\mu\nu}$ ), respectively. Then, remembering that  $\dot{\rho}_m + 3H(\rho_m + p_m) = 0$ , we have that the covariant derivative of the total energy-momentum tensor is

$$\nabla_\mu T^{\mu\nu} = \frac{\rho}{F+G} \left( \frac{dF}{d\phi} \dot{\phi} + \frac{dG}{d\chi} \dot{\chi} \right). \quad (17)$$

By observing (15) and (16), we note that their first terms on the right-hand sides represent the energy exchange between the fields  $\phi$  and  $\chi$  and their second terms on the right-hand sides describe the energy exchange among the scalar fields and gravitational field. From (17) one concludes that if  $F$  and  $G$  are constants,  $\nabla_\mu T^{\mu\nu} = 0$ , meaning that when the coupling is minimal there is no energy exchange among the scalar fields and gravitational field. If this is the case, there exists an energy exchange only between the scalar fields, and consequently the total energy related to the components of the Universe is conserved.

## 2.2. Couplings and potentials from the Noether symmetry

By starting from a general action, we can restrict the forms of the undefined couplings and potentials through the requirement of mathematical proprieties for the Lagrangian, such as symmetries. The symmetries may generate some formal suggestions for the possible forms of the undefined functions. In this work we will require that the Lagrangian of the general model satisfies the Noether symmetry, which provides a conserved quantity associated with the dynamical system. Interesting results may arise from the Noether symmetry approach, as can be found in the works [26, 27, 28, 29, 30].

A Noether symmetry for a given Lagrangian of the form  $\mathcal{L} = \mathcal{L}(q_i, \dot{q}_i)$  exists if the condition  $L_{\mathbf{X}}\mathcal{L} = \mathbf{X}\mathcal{L} = 0$  is satisfied, with  $L_{\mathbf{X}}$  designating the Lie derivative with respect to the vector field  $\mathbf{X}$  defined by

$$\mathbf{X} = \alpha_i \frac{\partial}{\partial q_i} + \frac{d\alpha_i}{dt} \frac{\partial}{\partial \dot{q}_i}, \quad (18)$$

where the  $\alpha_i$ 's are functions of the generalized coordinates  $q_i$ . The conserved quantity associated with the Noether symmetry generated by  $\mathbf{X}$  is given by

$$M_0 = \alpha_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i}. \quad (19)$$

The condition of existence for the Noether symmetry  $L_{\mathbf{X}}\mathcal{L} = \mathbf{X}\mathcal{L} = 0$  is applied to the point-like Lagrangian (6), with the vector field  $\mathbf{X}$  defined for our problem as follows

$$\mathbf{X} = \alpha \frac{\partial}{\partial a} + \beta \frac{\partial}{\partial \phi} + \gamma \frac{\partial}{\partial \chi} + \frac{\partial \alpha}{\partial t} \frac{\partial}{\partial \dot{a}} + \frac{\partial \beta}{\partial t} \frac{\partial}{\partial \dot{\phi}} + \frac{\partial \gamma}{\partial t} \frac{\partial}{\partial \dot{\chi}}, \quad (20)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are functions of  $(a, \phi, \chi)$ . In this case we obtain the following coupled system of differential equations

$$(F + G)\left(\alpha + 2a\frac{\partial\alpha}{\partial a}\right) + a\frac{dF}{d\phi}\left(\beta + a\frac{\partial\beta}{\partial a}\right) + a\frac{dG}{d\chi}\left(\gamma + a\frac{\partial\gamma}{\partial a}\right) = 0, \quad (21)$$

$$3\alpha - 12\frac{dF}{d\phi}\frac{\partial\alpha}{\partial\phi} + 2a\frac{\partial\beta}{\partial\phi} = 0, \quad (22)$$

$$3\alpha - 12\frac{dG}{d\chi}\frac{\partial\alpha}{\partial\chi} + 2a\frac{\partial\gamma}{\partial\chi} = 0, \quad (23)$$

$$a\beta\frac{d^2F}{d\phi^2} + \left(2\alpha + a\frac{\partial\alpha}{\partial a} + a\frac{\partial\beta}{\partial\phi}\right)\frac{dF}{d\phi} + a\frac{\partial\gamma}{\partial\phi}\frac{dG}{d\chi} + 2\frac{\partial\alpha}{\partial\phi}(F + G) - \frac{a^2}{6}\frac{\partial\beta}{\partial a} = 0, \quad (24)$$

$$a\gamma\frac{d^2G}{d\chi^2} + \left(2\alpha + a\frac{\partial\alpha}{\partial a} + a\frac{\partial\gamma}{\partial\chi}\right)\frac{dG}{d\chi} + a\frac{\partial\beta}{\partial\chi}\frac{dF}{d\phi} + 2\frac{\partial\alpha}{\partial\chi}(F + G) - \frac{a^2}{6}\frac{\partial\gamma}{\partial a} = 0, \quad (25)$$

$$\frac{\partial\alpha}{\partial\phi}\frac{dG}{d\chi} + \frac{\partial\alpha}{\partial\chi}\frac{dF}{d\phi} - \frac{a}{6}\left(\frac{\partial\beta}{\partial\chi} + \frac{\partial\gamma}{\partial\phi}\right) = 0, \quad (26)$$

$$3\alpha(V + W) + a\beta\left(\frac{dV}{d\phi} + \frac{\partial W}{\partial\phi}\right) + a\gamma\frac{\partial W}{\partial\chi} = 0. \quad (27)$$

The solution of the coupled system of differential equations (21)-(27) is not unique and the several solutions are found in Tables 1 and 2 which contain all sets of functions  $\alpha, \beta, \gamma, F, G, V, W$ , where the quantities  $\alpha_0, \beta_0, \gamma_0, F_0, F_0^1, G_0, G_0^1, V_0, W_0$  are constants and  $K = \beta_0/\gamma_0$ . We have looked for solutions which always furnish for the function  $W$  an expression different from  $(0, \text{constant}, f(\phi), g(\chi))$  in order to guarantee an interaction between the fields  $\phi$  and  $\chi$ .

	$\alpha$	$\beta$	$\gamma$	$F$	$G$	$V$	$W$
I	$\alpha_0 a$	$-3\alpha_0 \phi/2$	$-3\alpha_0 \chi/2$	$F_0 \phi^2$	$G_0 \chi^2$	$0, V_0 \phi^2$	$f(\chi/\phi) \phi^2$
II	$\alpha_0 a$	$-3\alpha_0 \phi/2$	$-3\alpha_0 \chi/2$	$F_0 \phi^2$	$0$	$0, V_0 \phi^2$	$f(\chi/\phi) \phi^2$
III	$\alpha_0 a$	$-3\alpha_0 \phi/2$	$-3\alpha_0 \chi/2$	$0$	$G_0 \chi^2$	$0, V_0 \phi^2$	$f(\chi/\phi) \phi^2$

**Table 1.** Solutions with  $(\alpha, \beta, \gamma) \neq 0$ .

One may observe from Table 2 that the general forms of  $W$  provided by the Noether symmetry allow the existence of sums which incorporate terms of the form  $f(\phi)$  – representing an additional term of self-interaction for the field  $\phi$  – namely,  $W = f(\phi) + g(\phi, \chi)$ . In this case, one must redefine the potentials in the energy density equations (11) and (12) and pressure equations (13) and (14) by writing  $W \rightarrow \overline{W} = W - f(\phi)$  and  $V \rightarrow \overline{V} = V + f(\phi)$ . So we take account the additional self-interaction term of the field  $\phi$ .

The case II in Table 1 with  $V = V_0 \phi^2$  is similar to the model analyzed in the work [17] and the case III in Table 2, when  $h = \phi^2 + \chi^2$ , is the model proposed in [12] with

	$\beta$	$\gamma$	$F$	$G$	$V$	$W$
I	$\beta_0\chi$	$-\beta_0\phi$	$F_0^1 + F_0\phi^2$	$G_0^1 + G_0\chi^2$	$\int(\frac{\phi}{\chi}\frac{\partial W}{\partial\chi} - \frac{\partial W}{\partial\phi})d\phi$	$W_0 \int h(\phi^2 + \chi^2)\chi d\chi$
II	$\beta_0\chi$	$-\beta_0\phi$	$F_0^1 + F_0\phi^2$	$G_0^1 + G_0\chi^2$	0, $V_0$	$W(\phi^2 + \chi^2)$
III	$\beta_0\chi$	$-\beta_0\phi$	$F_0$	$G_0$	$\int(\frac{\phi}{\chi}\frac{\partial W}{\partial\chi} - \frac{\partial W}{\partial\phi})d\phi$	$W_0 \int h(\phi^2 + \chi^2)\chi d\chi$
IV	$\beta_0\chi$	$-\beta_0\phi$	$F_0$	$G_0$	0, $V_0$	$W(\phi^2 + \chi^2)$
V	$\beta_0$	$\gamma_0$	$F_0^1 + F_0\phi$	$G_0^1 - K F_0\chi$	0, $V_0$	$W(\phi - K\chi)$
VI	$\beta_0$	$\gamma_0$	$F_0$	$G_0$	0, $V_0$	$W(\phi - K\chi)$

**Table 2.** Solutions with  $\alpha = 0$  and  $(\beta, \gamma) \neq 0$ .

Cases	$M_0$
I–III Tab 1	$\frac{3}{2}\alpha_0 a^3 \left\{ 2H \left[ 4(F + G) - 3 \left( \phi \frac{dF}{d\phi} + \chi \frac{dG}{d\chi} \right) \right] + \left( 4 \frac{dF}{d\phi} + \phi \right) \dot{\phi} + \left( 4 \frac{dG}{d\chi} + \chi \right) \dot{\chi} \right\}$
I–IV Tab 2	$\beta_0 a^3 \left\{ 6H \left( \chi \frac{dF}{d\phi} - \phi \frac{dG}{d\chi} \right) + \phi \dot{\chi} - \chi \dot{\phi} \right\}$
V–VI Tab 2	$\gamma_0 a^3 \left\{ 6H \left( K \frac{dF}{d\phi} + \frac{dG}{d\chi} \right) - K \dot{\phi} - \dot{\chi} \right\}$

**Table 3.** Conserved quantities.

$V = W_0\phi^4$  but with an additional self-interaction term of the form  $W_0\chi^4$ . The models of the works [12, 17] are particular cases of the one denoted by I in Table 2.

From the equation (19) we can write the conserved quantities associated with the cases in Tables 1 and 2. They are summarized in Table 3.

### 2.3. Solutions of the field equations

Due to the interaction between the fields  $\phi$  and  $\chi$  and the presence of a common matter field in the action (1), the field equations become more complicated than in the case with two non-interacting scalar fields and without a common matter field [28]. Hence, the search for numerical solutions of the system (7)-(10) for the most general cases in Tables 1 and 2 will be performed.

Let us transform the derivatives with respect to time in the equations (7)-(10) into derivatives with respect to red-shift through the relationships

$$z = \frac{1}{a} - 1, \quad \frac{d}{dt} = -H(1+z)\frac{d}{dz}, \quad (28)$$

and divide all the equations by  $\rho_0$  – the total energy density of the Universe at the present time. Hence, one obtains from the equations (7)-(10) the following system of coupled differential equations

$$4\widetilde{H}\widetilde{H}'(1+z)(F+G) = \widetilde{\rho} + \widetilde{p}, \quad (29)$$

$$\begin{aligned} &\widetilde{H}^2(1+z)^2\phi'' + \widetilde{H} \left[ \widetilde{H}'(1+z) - 2\widetilde{H} \right] \left[ (1+z)\phi' + 6\frac{dF}{d\phi} \right] \\ &+ \frac{d\widetilde{V}}{d\phi} + \frac{\partial\widetilde{W}}{\partial\phi} = 0, \end{aligned} \quad (30)$$

$$\widetilde{H}^2(1+z)^2\chi'' + \widetilde{H}[\widetilde{H}'(1+z) - 2\widetilde{H}]\left[(1+z)\chi' + 6\frac{dG}{d\chi}\right] + \frac{\partial\widetilde{W}}{\partial\chi} = 0, \quad (31)$$

by taking into account the equation (10). Above, the prime represents the derivative with respect to  $z$  and the following dimensionless quantities were introduced:  $\tilde{\rho} = \rho/\rho_0 = \rho_m/\rho_0 + \rho_\phi/\rho_0 + \rho_\chi/\rho_0 = \tilde{\rho}_m + \tilde{\rho}_\phi + \tilde{\rho}_\chi$ ,  $\tilde{p} = p/\rho_0 = p_\phi/\rho_0 + p_\chi/\rho_0 = \tilde{p}_\phi + \tilde{p}_\chi$ ,  $\widetilde{H} = H/\sqrt{\rho_0}$ ,  $\widetilde{V} = V/\rho_0$ , and  $\widetilde{W} = W/\rho_0$ . Furthermore, the dimensionless energy densities and pressures read

$$\tilde{\rho}_m = \tilde{\rho}_m^0(1+z)^3, \quad (32)$$

$$\tilde{\rho}_\phi = \frac{\widetilde{H}^2(1+z)^2\phi'^2}{2} + \widetilde{V} + 6\widetilde{H}^2(1+z)\frac{dF}{d\phi}\phi', \quad (33)$$

$$\tilde{\rho}_\chi = \frac{\widetilde{H}^2(1+z)^2\chi'^2}{2} + \widetilde{W} + 6\widetilde{H}^2(1+z)\frac{dG}{d\chi}\chi', \quad (34)$$

$$\begin{aligned} \tilde{p}_\phi = & \frac{\widetilde{H}^2(1+z)^2\phi'^2}{2} - \widetilde{V} + 2\widetilde{H}(1+z)\left\{\widetilde{H}(1+z)\left(\frac{d^2F}{d\phi^2}\phi'^2 + \frac{dF}{d\phi}\phi''\right)\right. \\ & \left.+ [\widetilde{H}'(1+z) - \widetilde{H}]\frac{dF}{d\phi}\phi'\right\}, \end{aligned} \quad (35)$$

$$\begin{aligned} \tilde{p}_\chi = & \frac{\widetilde{H}^2(1+z)^2\chi'^2}{2} - \widetilde{W} + 2\widetilde{H}(1+z)\left\{\widetilde{H}(1+z)\left(\frac{d^2G}{d\chi^2}\chi'^2 + \frac{dG}{d\chi}\chi''\right)\right. \\ & \left.+ [\widetilde{H}'(1+z) - \widetilde{H}]\frac{dG}{d\chi}\chi'\right\}. \end{aligned} \quad (36)$$

Our aim is to use the solutions given in Tables 1 and 2 in order to describe the dark sector as an interacting structure. Then we consider that the fields  $\phi$  and  $\chi$  correspond to the dark energy and dark matter fields, respectively. For this choice we have to require certain features for each field: (i) the field  $\phi$  must have a negative pressure in the late time and its energy density composes the most part of the total energy density of the Universe at the present time; (ii) the field  $\chi$  has a small positive pressure in comparison to the pressure modulus of the dark energy and its energy density still represents a considerable fraction of the total energy density of the Universe at the present time.

To satisfy the above requirements we will use the initial conditions for the system (29)-(31) which match the astronomical data. At  $z = 0$  one introduces the quantities  $\tilde{\rho}_m(0) = \rho_m^0/\rho_0 = \Omega_m^0$ ,  $\tilde{\rho}_\phi(0) = \rho_\phi^0/\rho_0 = \Omega_\phi^0$  and  $\tilde{\rho}_\chi(0) = \rho_\chi^0/\rho_0 = \Omega_\chi^0$ , where  $\Omega_i^0$  denotes the value of the density parameter of each component at the present time whereas  $\Omega_0 = \Omega_m^0 + \Omega_\phi^0 + \Omega_\chi^0$  refers to the total density parameter. The values of the density parameters adopted here are:  $\Omega_m^0 = 0.05$ ,  $\Omega_\phi^0 = 0.72$  and  $\Omega_\chi^0 = 0.23$  (see e.g. reference [31]). Further, in agreement with the requirement (i) one has that  $\phi'(0)^2 \ll 1$ , which means that the field  $\phi$  varies very slowly in the late time, i.e.,  $\phi'(0) = \epsilon$ , with  $\epsilon$  very small. From (33) and this last condition it follows that  $\widetilde{V}(0) \approx \Omega_\phi^0$  and one may obtain the initial condition for  $\phi$ . Once  $\phi(0)$  is fixed, one may determine the initial condition for  $\chi$  from the necessity that the coupling has the present value  $1/2$ , i.e.,  $F(0) + G(0) = 1/2$ . Satisfying the requirement II by the condition  $\tilde{\rho}_\chi(0) = \Omega_\chi^0$ , and since one knows  $\chi(0)$ , from (34) we may determine the initial condition for  $\chi'$ . The part of the requirement (ii)



that is related to the value of  $p_\chi$  can be satisfied through the adjustments of the constants that appear in the functions of the couplings and potentials. From (10) we have for the Hubble parameter the initial condition  $\widetilde{H}(0) = \sqrt{\Omega_0/6[F(0) + G(0)]} = 1/\sqrt{3}$ . To sum up we have:

- (a)  $\widetilde{H}(0) = \frac{1}{\sqrt{3}}$ ;
- (b)  $\widetilde{V}(0) = \Omega_\phi^0$  determines  $\phi(0)$ ,  $\phi'(0)^2 \ll 1$ ;
- (c)  $G(0) = \frac{1}{2} - F(0)$  determines  $\chi(0)$ ;
- (d)  $\chi'(0)^2 + 6\widetilde{W}(0) + 12\chi'(0)\frac{dG}{d\chi}\Big|_{z=0} = 6\Omega_\chi^0$  determines  $\chi'(0)$ .

For the case I in Table 1 we take

$$V = V_0\phi^2, \quad f\left(\frac{\chi}{\phi}\right) = \frac{\chi}{\phi} \quad \text{which implies} \quad W = W_0\phi\chi. \quad (37)$$

By using the above equations, the initial conditions are:

$$\begin{aligned} \phi(0) &= \sqrt{\frac{0.72}{\widetilde{V}_0}}, \quad \chi(0) = \sqrt{\frac{1/2 - F_0\phi(0)^2}{G_0}}, \\ \chi'(0) &= \sqrt{6[0.23 + 24G_0^2\chi(0)^2 - \widetilde{W}_0\phi(0)\chi(0)] - 12G_0\chi(0)}, \end{aligned}$$

where

$$F_0 \leq \frac{1}{2\phi(0)^2} \quad \text{and} \quad \widetilde{W}_0 \leq \frac{0.23/\chi(0) + 24G_0^2\chi(0)}{\phi(0)}. \quad (38)$$

For this case we have adopted the following values for the fixed constants in the numerical computations:  $F_0 = -0.002366$ ,  $G_0 = 0.04651$ ,  $V_0 = 0.01001$  and  $W_0 = 0.01856$ .

Now, for the case I in Table 2, by considering  $F_0^1 = G_0^1 = 0$  without loss of generality, one takes

$$h(\phi^2 + \chi^2) = \phi^2 + \chi^2 \quad \text{which implies} \quad W = W_0(\chi^4 + 2\phi^2\chi^2), \quad V = W_0\phi^4,$$

with  $W_0$  from Table 2 replaced by  $4W_0$ .

For this case one has the initial conditions:

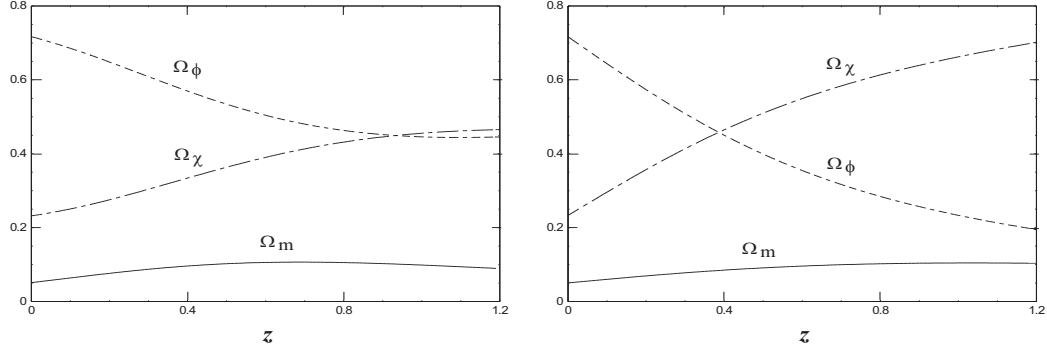
$$\begin{aligned} \phi(0) &= \left(\frac{0.72}{\widetilde{W}_0}\right)^{1/4}, \quad \chi(0) = \sqrt{\frac{1/2 - F_0\phi(0)^2}{G_0}}, \\ \chi'(0) &= \sqrt{6[0.23 + (24G_0^2 - \widetilde{W}_0)\chi(0)^4 - 2\widetilde{W}_0\phi(0)^2\chi(0)^2] - 12G_0\chi(0)}, \end{aligned}$$

where

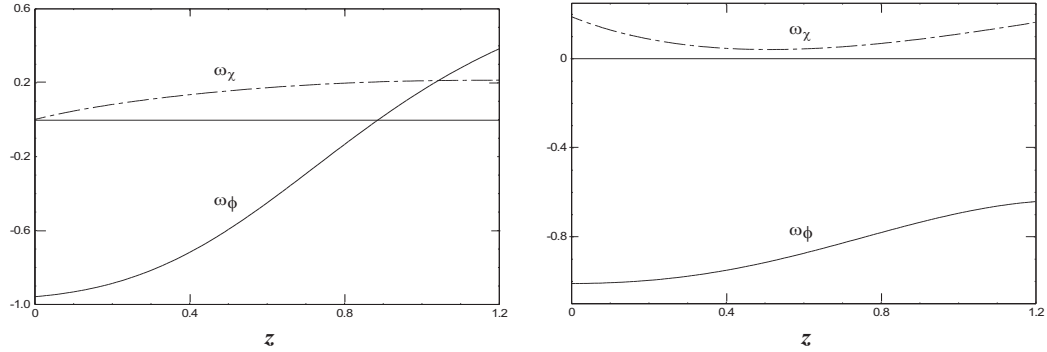
$$F_0 \leq \frac{1}{2\phi(0)^2} \quad \text{and} \quad \widetilde{W}_0 \leq \frac{0.23 + 24G_0^2\chi(0)^4}{\chi(0)^4 + 2\phi(0)^2\chi(0)^2}. \quad (39)$$

In this case we have taken for the fixed constants the values:  $F_0 = 0.2064$ ,  $G_0 = 0.03333$ , and  $W_0 = 0.1250$ .

In Figure 1 are represented the density parameters of the common matter, dark energy and dark matter for the cases 1 and 2 in the left and right frames, respectively. From this figure we can observe the evident difference of the increase of the density

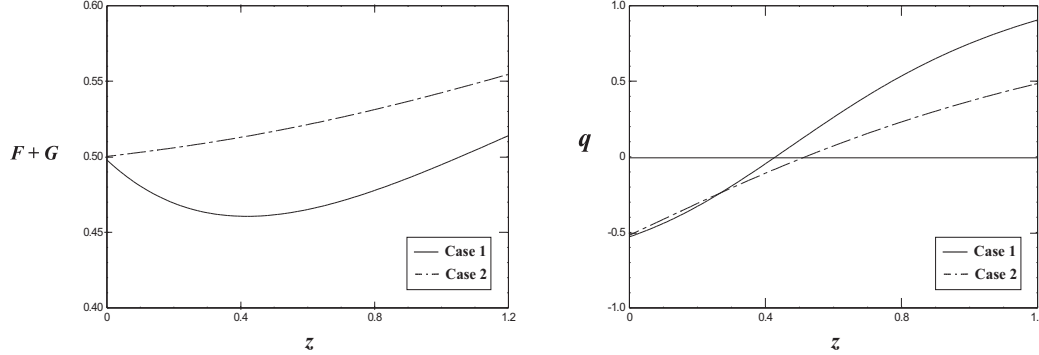


**Figure 1.** Density parameters of the common matter and scalar fields as functions of the red-shift  $z$ . Left frame: case 1; right frame: case 2. .



**Figure 2.** Ratio of the pressure and energy density of the scalar fields as functions of the red-shift  $z$ . Left frame: case 1; right frame: case 2.

parameter of the quintessence with the red-shift and the corresponding decrease of the density parameter of the dark matter for the two cases. Since the gravitational coupling has quadratic forms in both cases, the different red-shift evolutions of the density parameters are determined uniquely by the interaction and self-interaction potentials of the fields, which are the responsible of the energy transfer between the scalar fields and among the scalar fields and gravitational field, as can be observed from (15) and (16). Then these energy transfer among the fields (scalar field - scalar field and gravitational field - scalar fields) have a definitive role in the variety of behaviors which can be produced by models with scalar fields, as can be seen from these two cases in Figure 1. This is definitely verified when one observes from Figure 1 that the common matter field, which is not coupled to the other fields, has a red-shift evolution of its density parameter quite identical in the two cases, meaning that it is just submitted to the dilution caused by the expansion of the Universe, which presents practically the same rate for the two cases. From these proprieties, an interacting dark sector could present more possibilities for the energy density evolution of the dark matter.



**Figure 3.** Left frame: effective coupling versus red-shift  $z$ ; right frame: deceleration parameter versus red-shift  $z$ . The straight line represents the case 1 and the dashed line the case 2.

The ratio of the pressure and energy density of the scalar fields  $\omega_\phi = p_\phi/\rho_\phi$  and  $\omega_\chi = p_\chi/\rho_\chi$  are plotted in Figure 2, where the left frame corresponds to the case 1 and the right frame the case 2. These figures show that the pressure relative to the energy density of the dark matter field is small in comparison to the one (in modulus) of the dark energy for both cases. But the dark matter pressure has a significant role in the determination of the epoch where the transition of a decelerated to an accelerated expansion of the Universe occurs, since its participation in the total composition of the Universe is significant. Observe that in the present day  $\omega_\phi \rightarrow -1$ , which corresponds to a cosmological constant.

In the left frame of Figure 3 it is plotted the effective coupling  $F+G$  and in the right frame the deceleration parameter  $q = \frac{1}{2} + \frac{3p}{2\rho}$ , for the cases 1 and 2. We can infer from Figure 3 that the variation of the effective gravitational coupling presents a small value around its present value  $F(0) + G(0) = 1/2$ . This variation is about 10 percent, meaning that the effective gravitational "constant" varies approximately 10 percent in the considered interval. The right frame of Figure 3 shows that there exists a small difference between the red-shifts of the transition from a decelerated to an accelerated regime for the two cases. For the cases 1 and 2 the values of the red-shift transitions are  $z_T = 0.43$  and  $z_T = 0.52$  whereas the present values of the deceleration parameter read  $q(0) = -0.53$  and  $q(0) = -0.52$ , respectively. These results are in good agreement with the observational data, namely,  $z_T = 0.74 \pm 0.18$  (from [32]) and  $q(0) = -0.46 \pm 0.13$  (from [33]).

### 3. Interacting canonical and non-canonical scalar fields

#### 3.1. General action and Noether symmetry

Now let us take an action where one scalar field is non-canonical and represented by  $\varphi$ , being a tachyon-type field, and the other is a canonical scalar field represented by  $\phi$

$$S = \int d^4x \sqrt{-g} \left\{ [F(\varphi) + G(\phi)] R - V(\varphi) \sqrt{1 - \partial_\mu \varphi \partial^\mu \varphi} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - W(\varphi, \phi) \right\} + S_m, \quad (40)$$

where  $F(\varphi)$  and  $G(\phi)$  represent generic  $C^2$  functions which describe the coupling of the scalar fields to the gravity,  $V(\varphi)$  is the self-interaction potential of the field  $\varphi$  and  $W(\varphi, \phi)$  describes the self-interaction of the field  $\phi$  and the interaction between the fields  $\varphi$  and  $\phi$ . As before, when  $F(\varphi) + G(\phi) \rightarrow 1/2$  we recover the Einstein coupling.

From the variation of the action (40) with respect to  $g_{\mu\nu}$  one obtains the modified Einstein's equations with the same form of (2)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{T_{\mu\nu}}{2(F + G)}, \quad (41)$$

being  $T_{\mu\nu} = T_{\mu\nu}^m + T_{\mu\nu}^\varphi + T_{\mu\nu}^\phi$  defined as follows

$$T_{\mu\nu}^m = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}, \quad (42)$$

$$T_{\mu\nu}^\varphi = V \left( \frac{\partial_\mu \varphi \partial_\nu \varphi}{\sqrt{1 - \partial_\theta \varphi \partial^\theta \varphi}} + g_{\mu\nu} \sqrt{1 - \partial_\theta \varphi \partial^\theta \varphi} \right) + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\theta \nabla^\theta) F, \quad (43)$$

$$T_{\mu\nu}^\phi = \partial_\mu \phi \partial_\nu \phi - \left( \frac{1}{2} \partial_\theta \phi \partial^\theta \phi - W \right) g_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\theta \nabla^\theta) G. \quad (44)$$

By considering again a flat FRW metric and the scalar fields homogeneous,  $\varphi = \varphi(t)$  and  $\phi = \phi(t)$ , with the common matter being a pressureless fluid, the point-like Lagrangian which follows from the action (40) reads

$$\begin{aligned} \mathcal{L} = & 6a\dot{a}^2(F + G) + 6a^2\dot{a} \left( \frac{dF}{d\varphi} \dot{\varphi} + \frac{dG}{d\phi} \dot{\phi} \right) \\ & + a^3 V \sqrt{1 - \dot{\varphi}^2} - a^3 \left( \frac{1}{2} \dot{\phi}^2 - W \right) + \rho_m^0. \end{aligned} \quad (45)$$

From the condition of existence for the Noether symmetry  $L_{\mathbf{X}} \mathcal{L} = \mathbf{X} \mathcal{L} = 0$  applied to the point-like Lagrangian (45), with the vector field  $\mathbf{X}$  now defined as

$$\mathbf{X} = \alpha \frac{\partial}{\partial a} + \beta \frac{\partial}{\partial \varphi} + \gamma \frac{\partial}{\partial \phi} + \frac{\partial \alpha}{\partial t} \frac{\partial}{\partial \dot{a}} + \frac{\partial \beta}{\partial t} \frac{\partial}{\partial \dot{\varphi}} + \frac{\partial \gamma}{\partial t} \frac{\partial}{\partial \dot{\phi}}, \quad (46)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are functions of  $(a, \varphi, \phi)$ , we obtain the following system of partial differential equations

$$(F + G) \left( \alpha + 2a \frac{\partial \alpha}{\partial a} \right) + a \frac{dF}{d\varphi} \left( \beta + a \frac{\partial \beta}{\partial a} \right) + a \frac{dG}{d\phi} \left( \gamma + a \frac{\partial \gamma}{\partial a} \right) = 0, \quad (47)$$

$$\frac{\partial \alpha}{\partial \varphi} \frac{dF}{d\varphi} = 0, \quad \frac{\partial \beta}{\partial a} = 0, \quad \frac{\partial \beta}{\partial \varphi} = 0, \quad \frac{\partial \beta}{\partial \phi} = 0, \quad (48)$$

	$\alpha$	$\beta$	$\gamma$	$F$	$G$	$V$	$W$
I	$\alpha_0 a$	$\beta_0$	$-3\alpha_0\phi/2$	$F_0 e^{-\mu\varphi}$	$G_0\phi^2$	$V_0 e^{-\mu\varphi}$	$f(\phi e^{\frac{\mu\varphi}{2}})e^{-\mu\varphi}$
II	$\alpha_0 a$	$\beta_0$	$-3\alpha_0\phi/2$	$F_0 e^{-\mu\varphi}$	0	$V_0 e^{-\mu\varphi}$	$f(\phi e^{\frac{\mu\varphi}{2}})e^{-\mu\varphi}$
III	$\alpha_0 a$	$\beta_0$	$-3\alpha_0\phi/2$	0	$G_0\phi^2$	$V_0 e^{-\mu\varphi}$	$f(\phi e^{\frac{\mu\varphi}{2}})e^{-\mu\varphi}$
IV	0	$\beta_0$	$\gamma_0$	$F_0^1 + F_0\varphi$	$G_0^1 - K F_0\phi$	$V_0$	$W(\varphi - K\phi)$
V	0	$\beta_0$	$\gamma_0$	$F_0$	$G_0$	$V_0$	$W(\varphi - K\phi)$

**Table 4.** Solutions.

Cases	$M_0$
I–III	$\frac{3}{2}\alpha_0 a^3 \left\{ 2H \left[ 4(F + G) + 3 \left( \frac{2}{\mu} \frac{dF}{d\varphi} - \phi \frac{dG}{d\phi} \right) \right] + \left( 4 \frac{dF}{d\varphi} - \frac{2V}{\mu \sqrt{1-\dot{\varphi}^2}} \right) \dot{\varphi} + \left( 4 \frac{dG}{d\phi} + \phi \right) \dot{\phi} \right\}$
IV–V	$\gamma_0 a^3 \left\{ 6H \left( K \frac{dF}{d\varphi} + \frac{dG}{d\phi} \right) - \frac{KV\dot{\varphi}}{\sqrt{1-\dot{\varphi}^2}} - \dot{\phi} \right\}$

**Table 5.** Conserved quantities.

$$3\alpha - 12 \frac{dG}{d\phi} \frac{\partial \alpha}{\partial \phi} + 2a \frac{\partial \gamma}{\partial \phi} = 0, \quad (49)$$

$$a\beta \frac{d^2 F}{d\varphi^2} + \left( 2\alpha + a \frac{\partial \alpha}{\partial a} + a \frac{\partial \beta}{\partial \varphi} \right) \frac{dF}{d\varphi} + a \frac{\partial \gamma}{\partial \varphi} \frac{dG}{d\phi} + 2 \frac{\partial \alpha}{\partial \varphi} (F + G) = 0, \quad (50)$$

$$a\gamma \frac{d^2 G}{d\phi^2} + \left( 2\alpha + a \frac{\partial \alpha}{\partial a} + a \frac{\partial \gamma}{\partial \phi} \right) \frac{dG}{d\phi} + a \frac{\partial \beta}{\partial \phi} \frac{dF}{d\varphi} + 2 \frac{\partial \alpha}{\partial \phi} (F + G) - \frac{a^2}{6} \frac{\partial \gamma}{\partial a} = 0, \quad (51)$$

$$\frac{\partial \alpha}{\partial \varphi} \frac{dG}{d\phi} + \frac{\partial \alpha}{\partial \phi} \frac{dF}{d\varphi} - \frac{a}{6} \left( \frac{\partial \beta}{\partial \phi} + \frac{\partial \gamma}{\partial \varphi} \right) = 0, \quad (52)$$

$$3\alpha V + a\beta \frac{dV}{d\varphi} = 0, \quad (53)$$

$$3\alpha W + a\beta \frac{\partial W}{\partial \varphi} + a\gamma \frac{\partial W}{\partial \phi} = 0. \quad (54)$$

The solution of the system (47)-(54) is not unique and the solutions that we found are given in Table 4 containing the sets of  $\alpha, \beta, \gamma, F, G, V, W$ , where  $\alpha_0, \beta_0, \gamma_0, F_0, F_0^1, G_0, G_0^1$  and  $V_0$  are constants and  $\mu = 3\alpha_0/\beta_0$  and  $K = \beta_0/\gamma_0$ . Here we also looked for solutions which present potentials of the form  $W \neq (0, \text{constant}, f(\varphi), g(\phi))$  in order to provide an interaction between the fields  $\varphi$  and  $\phi$ . And from (19) we have the respective conserved quantities, which are given in Table 5.

It is interesting to observe that the solution I generalizes the model analyzed in the work [26].

### 3.2. Field equations and energy exchange

From the Euler-Lagrange equations applied to the Lagrangian (45) for  $a$ ,  $\varphi$  and  $\phi$ , respectively, one has

$$2(F+G)(2\dot{H}+3H^2) - V\sqrt{1-\dot{\varphi}^2} + \frac{1}{2}\dot{\phi}^2 - W + 2\left(\frac{dF}{d\varphi}\ddot{\varphi} + 2H\frac{dF}{d\varphi}\dot{\varphi} + \frac{d^2F}{d\varphi^2}\dot{\varphi}^2\right) + 2\left(\frac{dG}{d\phi}\ddot{\phi} + 2H\frac{dG}{d\phi}\dot{\phi} + \frac{d^2G}{d\phi^2}\dot{\phi}^2\right) = 0, \quad (55)$$

$$\frac{\ddot{\varphi}}{1-\dot{\varphi}^2} + 3H\dot{\varphi} + \frac{1}{V}\frac{dV}{d\varphi} + \left[\frac{\partial W}{\partial\varphi} - 6(\dot{H}+2H^2)\frac{dF}{d\varphi}\right]\frac{\sqrt{1-\dot{\varphi}^2}}{V} = 0, \quad (56)$$

$$\ddot{\phi} + 3H\dot{\phi} - 6(\dot{H}+2H^2)\frac{dG}{d\phi} + \frac{\partial W}{\partial\phi} = 0. \quad (57)$$

As in the previous section, by imposing that the energy function  $E_{\mathcal{L}} = \frac{\partial\mathcal{L}}{\partial\dot{a}}\dot{a} + \frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\dot{\varphi} + \frac{\partial\mathcal{L}}{\partial\dot{\phi}}\dot{\phi} - \mathcal{L}$  associated with the Lagrangian (45) is null, it follows

$$6(F+G)H^2 - \frac{\rho_m^0}{a^3} - \frac{V}{\sqrt{1-\dot{\varphi}^2}} + 6H\frac{dF}{d\varphi}\dot{\varphi} - \frac{1}{2}\dot{\phi}^2 - W + 6H\frac{dG}{d\phi}\dot{\phi} = 0. \quad (58)$$

From the equations (55)-(58) one defines  $\rho = \rho_m + \rho_\varphi + \rho_\phi$  and  $p = p_\varphi + p_\phi$ , with their forms given by

$$\rho_\varphi = \frac{V}{\sqrt{1-\dot{\varphi}^2}} - 6H\frac{dF}{d\varphi}\dot{\varphi}, \quad (59)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + W - 6H\frac{dG}{d\phi}\dot{\phi}, \quad (60)$$

$$p_\varphi = -V\sqrt{1-\dot{\varphi}^2} + 2\left(\frac{dF}{d\varphi}\ddot{\varphi} + 2H\frac{dF}{d\varphi}\dot{\varphi} + \frac{d^2F}{d\varphi^2}\dot{\varphi}^2\right), \quad (61)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - W + 2\left(\frac{dG}{d\phi}\ddot{\phi} + 2H\frac{dG}{d\phi}\dot{\phi} + \frac{d^2G}{d\phi^2}\dot{\phi}^2\right), \quad (62)$$

in agreement with the definitions of the energy-momentum tensors (43) and (44).

By using the definitions of the energy densities (59) and (60) and their respective pressures (61) and (62), one has

$$\dot{\rho}_\varphi + 3H(\rho_\varphi + p_\varphi) = -\frac{\partial W}{\partial\varphi}\dot{\varphi} + \frac{(dF/d\varphi)\dot{\varphi}}{F+G}\rho, \quad (63)$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = \frac{\partial W}{\partial\phi}\dot{\phi} + \frac{(dG/d\phi)\dot{\phi}}{F+G}\rho, \quad (64)$$

where (56), (57) and (58) were used for the simplifications. Then, proceeding as in the section 2, we have the covariant derivative of the total energy-momentum tensor

$$\nabla_\mu T^{\mu\nu} = \frac{\rho}{F+G}\left(\frac{dF}{d\varphi}\dot{\varphi} + \frac{dG}{d\phi}\dot{\phi}\right), \quad (65)$$

which has the same form of (17).

From these results, we will consider the interacting dark sector model as before: one takes the field  $\varphi$  to represent the dark energy and the field  $\phi$  to represent the dark matter. And following the astronomical constrains: (i) The field  $\varphi$  composes the most

part of the total energy density and has an expressive negative pressure in the late time; (ii) The field  $\phi$  has a small positive pressure and its energy density represents a considerable fraction of the total energy density in the present.

### 3.3. Cosmological solutions

Heaving in view the difficulties of integration, we will search for numerical solutions for the system (55)-(58). In order to analyze the cosmological scenarios that these models can describe, the solutions for some cases in Table 4 will be considered.

Let us firstly transform the derivatives with respect to time in the system (55)-(58) into derivatives with respect to red-shift. In addition, by substituting  $H^2$  from the equation (58) into the equation (55), we obtain the following final system of coupled differential equations to solve

$$4\widetilde{H}\widetilde{H}'(1+z)(F+G) = \widetilde{\rho} + \widetilde{p}, \quad (66)$$

$$\frac{\widetilde{H}^2(1+z)^2\widetilde{\varphi}'' + \widetilde{H}(1+z)[\widetilde{H}'(1+z) + \widetilde{H}]\widetilde{\varphi}'}{1 - \widetilde{H}^2(1+z)^2\widetilde{\varphi}'^2} + \frac{1}{\widetilde{V}}\frac{d\widetilde{V}}{d\widetilde{\varphi}} - 3\widetilde{H}^2(1+z)\widetilde{\varphi}' + \left\{6\widetilde{H}[\widetilde{H}'(1+z) - 2\widetilde{H}]\frac{dF}{d\widetilde{\varphi}} + \frac{\partial\widetilde{W}}{\partial\widetilde{\varphi}}\right\}\frac{\sqrt{1 - \widetilde{H}^2(1+z)^2\widetilde{\varphi}'^2}}{\widetilde{V}} = 0, \quad (67)$$

$$\widetilde{H}^2(1+z)^2\phi'' + \widetilde{H}[\widetilde{H}'(1+z) - 2\widetilde{H}]\left[(1+z)\phi' + 6\frac{dG}{d\phi}\right] + \frac{\partial\widetilde{W}}{\partial\phi} = 0, \quad (68)$$

with the line representing derivative with respect to  $z$ , where  $\widetilde{H} = H/\sqrt{\rho_0}$ ,  $\widetilde{\varphi} = \sqrt{\rho_0}\varphi$ ,  $\widetilde{V} = V/\rho_0$ ,  $\widetilde{W} = W/\rho_0$ ,  $\widetilde{\rho} = \rho/\rho_0 = \rho_m/\rho_0 + \rho_\varphi/\rho_0 + \rho_\phi/\rho_0 = \widetilde{\rho}_m + \widetilde{\rho}_\varphi + \widetilde{\rho}_\phi$  and  $\widetilde{p} = p/\rho_0 = p_\varphi/\rho_0 + p_\phi/\rho_0 = \widetilde{p}_\varphi + \widetilde{p}_\phi$ , which are dimensionless quantities. The energy densities and pressures are now given by

$$\widetilde{\rho}_m = \widetilde{\rho}_m^0(1+z)^3, \quad (69)$$

$$\widetilde{\rho}_\varphi = \frac{\widetilde{V}}{\sqrt{1 - \widetilde{H}^2(1+z)^2\widetilde{\varphi}'^2}} + 6\widetilde{H}^2(1+z)\frac{dF}{d\widetilde{\varphi}}\widetilde{\varphi}', \quad (70)$$

$$\widetilde{\rho}_\phi = \frac{\widetilde{H}^2(1+z)^2\phi'^2}{2} + \widetilde{W} + 6\widetilde{H}^2(1+z)\frac{dG}{d\phi}\phi', \quad (71)$$

$$\begin{aligned} \widetilde{p}_\varphi = & 2\widetilde{H}(1+z)\left\{\widetilde{H}(1+z)\left(\frac{d^2F}{d\widetilde{\varphi}^2}\widetilde{\varphi}'^2 + \frac{dF}{d\widetilde{\varphi}}\widetilde{\varphi}''\right) + [\widetilde{H}'(1+z) - \widetilde{H}]\frac{dF}{d\widetilde{\varphi}}\widetilde{\varphi}'\right\} \\ & - \widetilde{V}\sqrt{1 - \widetilde{H}^2(1+z)^2\widetilde{\varphi}'^2}, \end{aligned} \quad (72)$$

$$\begin{aligned} \widetilde{p}_\phi = & 2\widetilde{H}(1+z)\left\{\widetilde{H}(1+z)\left(\frac{d^2G}{d\phi^2}\phi'^2 + \frac{dG}{d\phi}\phi''\right) + [\widetilde{H}'(1+z) - \widetilde{H}]\frac{dG}{d\phi}\phi'\right\} \\ & + \frac{\widetilde{H}^2(1+z)^2\phi'^2}{2} - \widetilde{W}. \end{aligned} \quad (73)$$

The requirements (i) and (ii) for the fields  $\phi$  and  $\varphi$  will be satisfied by using the initial conditions for the system (66)-(68) determined from the astronomical data, as it is done in the canonical - canonical model. From the requirement I we have that  $\widetilde{\varphi}'(0)^2 \ll 1$ , that is, the field  $\varphi$  is varying very slowly in the late time (the

same consideration of Section 2). This condition and (70) imply in the relation  $\tilde{V}(0) \approx \Omega_\varphi^0$ . Remembering that the gravitational coupling must have the value  $1/2$  in the present,  $F(0) + G(0) = 1/2$ , the equation (58) furnishes the initial condition  $\tilde{H}(0) = \sqrt{\Omega_0/6[F(0) + G(0)]} = 1/\sqrt{3}$  to the Hubble parameter, just as it was in the first case. All these relations will be employed to perform comparisons to the observational data.

From now on, we will analyze the cases I and V from Table 4, which represent interacting models non-minimally and minimally coupled to the gravity, respectively.

For the case I, we have chosen

$$f\left(\phi e^{\frac{\mu\varphi}{2}}\right) = \frac{e^{-\frac{\mu\varphi}{2}}}{\phi}, \quad \text{which implies} \quad W = W_0 \frac{e^{-\frac{3\mu\varphi}{2}}}{\phi}. \quad (74)$$

For these functions, one may determine that the initial conditions are given by:

$$\begin{aligned} \tilde{\varphi}(0) &= \frac{\ln[\tilde{V}_0/0.72]}{\tilde{\mu}}, & \phi(0) &= \sqrt{\frac{1/2 - F_0 e^{-\tilde{\mu}\tilde{\varphi}(0)}}{G_0}}, \\ \phi'(0) &= \sqrt{6 \left[ 0.23 + 24G_0^2\phi(0)^2 - \tilde{W}_0\phi(0)^{-1}e^{-\frac{3\tilde{\mu}\tilde{\varphi}(0)}{2}} \right]} - 12G_0\phi(0), \end{aligned}$$

where  $\tilde{\mu} = \mu/\sqrt{\rho_0}$  and

$$F_0 \leq \frac{e^{\tilde{\mu}\tilde{\varphi}(0)}}{2}, \quad \tilde{W}_0 \leq [0.23 + 24G_0^2\phi(0)^2]\phi(0)e^{\frac{3\tilde{\mu}\tilde{\varphi}(0)}{2}}. \quad (75)$$

For the derivative of the field  $\varphi$  at  $z = 0$  we have chosen  $\tilde{\varphi}'(0) = 10^{-6}$  and the following values have been adopted for the fixed constants:  $F_0 = -8.5 \times 10^{-3}$ ;  $G_0 = 6.9 \times 10^{-3}$ ;  $\tilde{V}_0 = 2.2 \times 10^{-3}$ ;  $\tilde{W}_0 = 1.9 \times 10^{-3}$ . Two values for the coefficient in the exponential term were adopted, namely,  $\tilde{\mu}_1 = 10^{-3}$  and  $\tilde{\mu}_2 = 10^{-2}$ .

For the case V we have considered that the pressure of the dark matter vanishes at  $z = 0$  and that the interaction potential of the scalar fields is given by

$$W(\varphi - \kappa\phi) = W_0 e^{-\xi(\varphi - \kappa\phi)}.$$

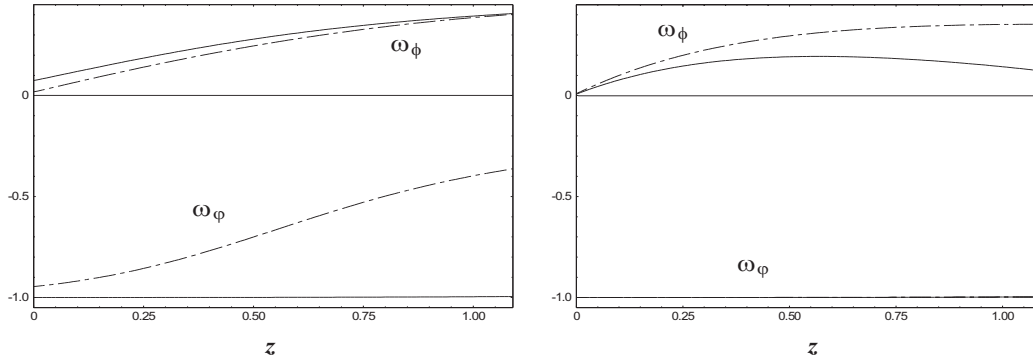
From the subtraction and sum of the equations (71) and (73) one obtains the initial conditions for  $\phi(0)$  and  $\phi'(0)$ , respectively,

$$\phi(0) = \frac{\tilde{\varphi}(0) + \ln(0.115/\tilde{W}_0)/\tilde{\xi}}{\tilde{\kappa}}, \quad \phi'(0) = \sqrt{0.69},$$

where  $\tilde{\xi} = \xi/\sqrt{\rho_0}$  and  $\tilde{\kappa} = \sqrt{\rho_0}\kappa$ . The initial conditions for  $\tilde{\varphi}(0)$  and  $\tilde{\varphi}'(0)$  are free and were chosen as  $\tilde{\varphi}(0) = 1.0$  and  $\tilde{\varphi}'(0) = 10^{-2}$ . For the fixed constants the following values were adopted:  $F_0 + G_0 = 1/2$ ;  $\tilde{V}_0 = 0.72$ ;  $\tilde{W}_0 = 10^{-2}$ ;  $\tilde{\kappa} = 0.5405$ . As in the previous case, two values for the coefficient in the exponential term were adopted, namely,  $\tilde{\xi}_1 = 4.9$  and  $\tilde{\xi}_2 = 4.45$ .

In Figure 4 are plotted the ratio of the pressure and energy density of the scalar fields,  $\omega_\varphi = p_\varphi/\rho_\varphi$  and  $\omega_\phi = p_\phi/\rho_\phi$ , where the left and right frames represent the cases I

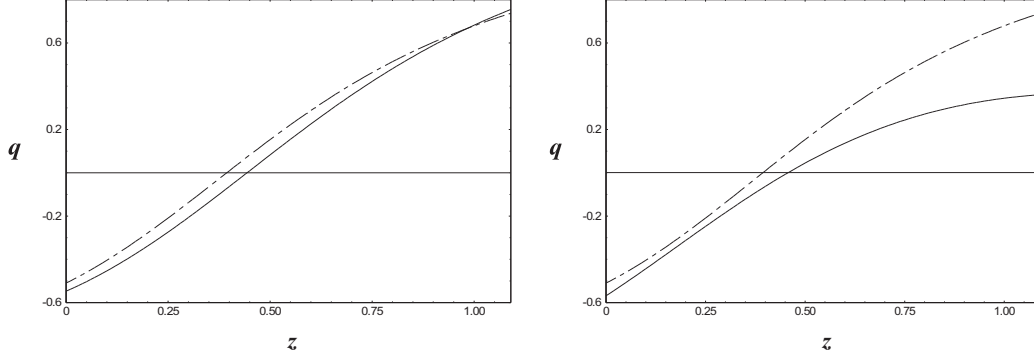




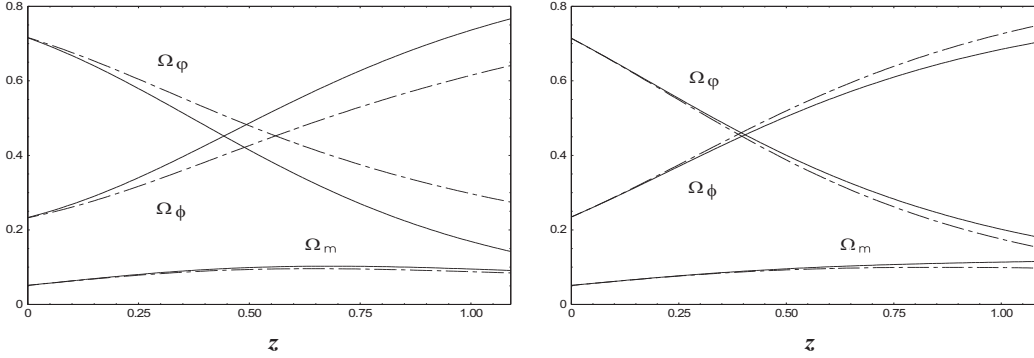
**Figure 4.** Left frame: the ratio of the pressure and energy density of the scalar fields for the case I, represented by the straight line for  $\tilde{\mu}_1 = 10^{-3}$  and by the dashed line for  $\tilde{\mu}_2 = 10^{-2}$ . Right frame: the ratio of the pressure and energy density of the scalar fields for the case V, represented by the straight line for  $\tilde{\xi}_1 = 4.900$  and by the dashed line for  $\tilde{\xi}_2 = 4.450$ .

and V, respectively. From this figure one can infer that when the parameter  $\tilde{\mu}$  is varied from  $10^{-3}$  to  $10^{-2}$  (case I) the ratio  $\omega_\varphi$  changes its red-shift evolution drastically. This behavior can be understood because – according to (63) – this ratio is related to the direct exchange of energy between the field  $\varphi$  and gravitational field. Then the behavior of the dark energy changes from a cosmological constant-type  $\omega_\varphi \approx -1$  for  $\tilde{\mu}_1 = 10^{-3}$  to a variable  $\omega_\varphi$  for  $\tilde{\mu}_2 = 10^{-2}$  as a consequence of the modification in the direct energy exchange with the gravitational field. However, the ratio  $\omega_\phi$  has a smooth variation when the values of the coefficient in the exponential term are changed. For the case V, when the parameter  $\tilde{\xi}$  is varied, one notes by observing the behavior of the ratio  $\omega_\phi$  that the dark matter suffers a significant influence, while the dark energy always has an approximated cosmological constant-type behavior, since a very small variation of the ratio  $\omega_\varphi$  occurs. Observe that in the case V there is no direct energy exchange with the gravitational field due to the conditions  $\{F, G\} = \text{constant}$ .

The deceleration parameter  $q = 1/2 + 3p/2\rho$  is represented in Figure 5, for the cases I (left frame) and V (right frame). The left frame of this figure shows us that the deceleration parameter exhibits a small modification when one varies the value of the coefficient  $\tilde{\mu}$  in the exponential term. However, one may infer that when  $\tilde{\mu}_2 = 10^{-2}$  and  $\omega_\varphi$  goes asymptotically to  $-1$  in the late time, the red-shift transition from a decelerated to a accelerated regime is smaller than that for  $\tilde{\mu}_1 = 10^{-3}$ . Indeed, in this last situation  $\omega_\varphi \approx -1$  in the whole evolution of the density parameter, which implies an earlier transition of regime. For the case V, one may observe that for  $\tilde{\xi}_2 = 4.45$  the red-shift transition is smaller than that for  $\tilde{\xi}_1 = 4.9$ . This can be understood by looking at Figure 4 again, where one can infer that  $\omega_\phi$  is larger for  $\tilde{\xi}_2 = 4.45$  than for  $\tilde{\xi}_1 = 4.9$ , meaning that in this situation the dark matter has a larger relative pressure, which contributes to retard the transition of regime. The values of the red-shift transition for the case I are:  $z_T = 0.45$  ( $\tilde{\mu}_1 = 10^{-3}$ ) and  $z_T = 0.40$  ( $\tilde{\mu}_2 = 10^{-2}$ ), whereas those for the case



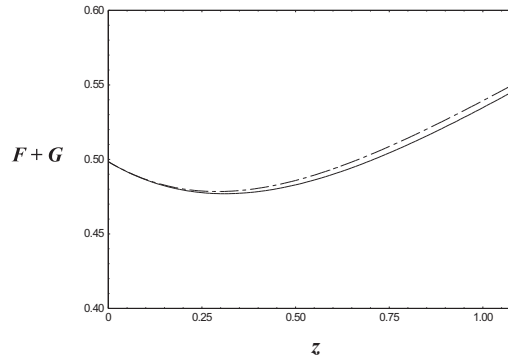
**Figure 5.** Left frame: deceleration parameter for the case I, represented by the straight line for  $\tilde{\mu}_1 = 10^{-3}$  and by the dashed line for  $\tilde{\mu}_2 = 10^{-2}$ . Right frame: deceleration parameter for the case V, represented by the straight line for  $\tilde{\xi}_1 = 4.900$  and by the dashed line for  $\tilde{\xi}_2 = 4.450$ .



**Figure 6.** Left frame: parameters of density for the case I, represented by the straight line for  $\tilde{\mu}_1 = 10^{-3}$  and by the dashed line for  $\tilde{\mu}_2 = 10^{-2}$ . Right frame: parameters of density for the case V, represented by the straight line for  $\tilde{\xi}_1 = 4.900$  and by the dashed line for  $\tilde{\xi}_2 = 4.450$ .

V are:  $z_T = 0.46$  ( $\tilde{\xi}_1 = 4.9$ ) and  $z_T = 0.40$  ( $\tilde{\xi}_2 = 4.45$ ). Furthermore, the values of the deceleration parameter  $q_0$  at  $z = 0$  for the case I are:  $q_0 = -0.55$  ( $\tilde{\mu}_1 = 10^{-3}$ ) and  $q_0 = -0.51$  ( $\tilde{\mu}_2 = 10^{-2}$ ), while those for the case V are:  $q_0 = -0.57$  ( $\tilde{\xi}_1 = 4.9$ ) and  $q_0 = -0.51$  ( $\tilde{\xi}_2 = 4.45$ ). In order to perform comparisons to the observational data, the recent observed values are  $z_T = 0.74 \pm 0.18$  (from [32]) for the red-shift transition and  $q_0 = -0.46 \pm 0.13$  (from [33]) for the deceleration parameter at  $z = 0$ . Hence, one may conclude that there exists a good agreement of the results with the observational data.

The density parameters of the common matter, dark energy and dark matter fields are represented in Figure 6 for the cases I (left frame) and V (right frame). From the left frame one observes that for  $\tilde{\mu}_2$  the density parameters of the dark energy and dark matter become equals earlier than for  $\tilde{\mu}_1$ , but its red-shift transition occurs at a smaller red-shift than that for  $\tilde{\mu}_1$ . This shows that the change of the behavior of  $\omega_\varphi$  caused by



**Figure 7.** Effective gravitational coupling for the case I, represented by the straight line for  $\tilde{\mu}_1 = 10^{-3}$  and by the dashed line for  $\tilde{\mu}_2 = 10^{-2}$ .

the change of the energy exchange with the gravitational field really is the responsible for a smaller red-shift transition in the case I. On the other hand, by observing the right frame, one notes that for  $\tilde{\xi}_1$  the density parameters of the dark energy and dark matter become equals earlier than for  $\tilde{\xi}_2$ . This reinforces the delay of the red-shift transition caused by a larger relative pressure of the dark matter for  $\tilde{\xi}_2$ .

The effective coupling  $F+G$  for the case I is plotted in Figure 7. One observes from this figure that the effective gravitational coupling has a small variation in comparison to its present value  $F(0)+G(0) = 1/2$ . There is a variation of less than 10% around the value  $1/2$ , and consequently the effective gravitational "constant" has its value changed about 10% in the considered interval. This result is similar to that of the canonical - canonical case.

#### 4. Conclusions

By applying the Noether symmetry approach we have restricted the possible functions of the undefined couplings and potentials of the general models to families of functions. Using this tool we could analyze the cosmological solutions of some particular interacting dark sector models, which correspond to potentials and couplings satisfying the symmetry condition for the general actions. Some of the resulting models from the symmetry condition generalize certain interacting dark sectors models that have appeared in the literature. The energy exchange which occurs among the fields (scalar field - scalar field and gravitational field - scalar fields) has strong influence on the behaviors described by models with scalar fields. An important verification is that non-minimal couplings can have a very significant influence on the evolution of the energy densities and pressures of the components of the Universe. The results for both general models (canonical - canonical and non-canonical - canonical) showed distinct ways to the energy density and pressure evolutions in similar regimes of expansion of the Universe. Further, both can reproduce a decelerated-accelerated regime, describing

the recent transition from a decelerated to an accelerated expansion in agreement with the observational data. Canonical - non-canonical models can reproduce behaviors very similar to that of the cosmological constant model to the late Universe (with respect to the ratio of the pressure and energy density of the dark energy field), but with the additional advantage of presenting more possible ways for the energy density evolution of the matter fields.

## References

- [1] Zwicky F 1933 *Helv. Phys. Acta* **6** 110
- [2] Freese K Fields B and Graff D 2000 *arXiv: astro-ph/0007.444*
- [3] Clowe D. *et al.* 2006 *Astrophys. J.* **648** L109
- [4] Massey R *et al.* 2007 *Nature* **445** 286
- [5] Riess A G *et al.* 1998 *Astron. J.* **116** 1009
- [6] Perlmutter S *et al.* 1999 *Astrophys. J.* **517** 565
- [7] Peebles P J E and Ratra B 2003 *Rev. Mod. Phys.* **75** 559
- [8] Szydlowski M, Kurek A and Krawieck A 2006 *Phys. Lett. B* **642** 171
- [9] Comelli D, Pietroni M and Riotto A 2003 *Phys. Letters B* **571** 115
- [10] Abdalla E, Abramo L R, Sodré Jr. L and Wang B 2009 *Phys. Letters B* **673** 107
- [11] Zhou J, Wang B, Pavón D and Abdalla E 2009 *Mod. Phys. Lett. A* **24** 1689
- [12] Hoffman M B 2003 *arXiv: astro-ph/0307350*.
- [13] Axenides M and Dimopoulos K 2004 *JCAP* **07** 010
- [14] de la Macorra A 2008 *JCAP* **01** 030
- [15] Chimento L P, Forte M and Kremer G M 2009 *Gen. Rel. Grav.* **41** 1125
- [16] Caldera-Cabral G and Maartens R 2009 *Phys. Review D* **79** 063518
- [17] Binder J B and Kremer G M 2006 *Gen. Rel. Grav.* **38** 857
- [18] Kremer G M 2007 *Gen. Rel. Grav.* **39** 965
- [19] Caldera-Cabral G, Maartens R and Schaefer B M 2009 *JCAP* **07** 027
- [20] Bagla J S, Jassal H K and Padmanabhan T 2003 *Phys. Rev. D* **67** 063504
- [21] Jassal H K 2004 *Pramana* **62** 757
- [22] Kremer G M and Alves D S M 2004 *Gen. Relativ. Grav.* **36** 2039
- [23] Das A, Gupta S, Saini T D and Kar S 2005 *Phys. Rev. D* **72** 043528
- [24] Panotopoulos G 2006 *arXiv: astro-ph/0606249*
- [25] Ren J and Meng X 2008 *arXiv: astro-ph/0610266*
- [26] de Souza R and Kremer G M 2009 *Class. Quantum Grav.* **26** 135008
- [27] Capozziello S and de Ritis R 1994 *Class. Quantum Grav.* **11** 107
- [28] Capozziello S and Lambiase G 2000 *Grav. Cosmol.* **6** 164
- [29] Capozziello S, Dunsby P K S, Piedipalumbo E and Rubano C 2007 *Astron. and Astroph.* **472** 51
- [30] de Souza R and Kremer G M 2008 *Class. Quantum Grav.* **25** 225006
- [31] Fukugita M and Peebles P J E 2004 *Astrophys. J.* **616** 643
- [32] Virey J M *et al.* 2005 *Phys. Rev. D* **72** 061302
- [33] Riess A G *et al.* 2004 *Astrophys. J.* **607** 665